

Alpha and Betas

A word about alpha and betas in investment returns

The Math

In mathematics, half the problem is giving things names. The investment management industry has borrowed a naïve model, applied it to very complicated problems and then expect to make sense of the results.

What is alpha and what are betas?

The terms alpha and beta come from the linear model of statistical modelling.

$$y_i = b x_i + a + e_i$$

Where y_i is the dependent variable, x_i is the independent variable, a is an intercept term and e_i is an error term which by construction has a mean value of zero.

The standard example is where y_i are the returns of a particular stock, x_i are the returns of the market (using some suitable stock index as proxy). The coefficient b is the beta and represents the systemic risk, the coefficient a is alpha which represents specific risk.

In order to make statistical inferences from the model a distribution needs to be assigned to the error term e_i . There is a theorem that says that subject to some assumptions about how e_i behaves, not only is it possible to estimate what the betas and alpha look like but we can make inferences from them.

The model is easily extended and generalized to :

$$y_i = \text{sum of } (b_j x_{ij}) + a + e_i$$

Where there is not one market factor but k of them. The

industry applies this model to hedge fund returns often with one factor, usually an equity index, and sometimes to several factors. Natural candidate factors are, bonds, yield curve shape, equity vol, swaption vol, credit spreads, interest rates, currencies.

Observations and comments:

An assumption is being made about the relationship between y and the x 's. If the assumptions are wrong, the betas and alpha measured are meaningless. The industry will sometimes apply the model to a credit manager, or a fixed income arbitrageur, or an asset based lender with equity market returns as an explanatory variable, despite the lack of causality.

There has to be sufficient data. The more complicated the model, the more data you need. Hedge funds publish monthly performance numbers. A manager with a 5 year track record has only got 60 data points. Having enough data is the first point. The data has also to be well behaved.

Proper estimation of betas and alpha require that the x 's have certain properties. One of them is that the x 's should not cluster, mean revert or converge. This is clearly a problem. Basically what the methodology requires, in simple terms, is that if you want to measure a manager's alpha, you need to have all sorts of market conditions from bull and bear trends to choppy sideways markets. It makes sense. A 5 X levered position in a rising market looks very much like alpha. What does one call a 0.5X levered position in a rising market? Negative alpha?

The two previous points suggest also that the data has to come from a sufficiently diverse set of states. Enough data and enough variation in data imply that a manager has to be tested over all phases of the cycle in their particular market. Ideally, the performance should be measured over several cycles.

The necessary conditions for meaningful inference make this methodology intractable for hedge fund analysis. Track records are rarely sufficiently long to include several iterations of the market cycle.

Comments about industry implementation:

Seeking to buy alpha is only relevant if one is willing to invest over sufficient cycles for the alpha to manifest.

Beta is cheap. Alpha is not priced. It may be expensive or not, but current performance fees are not directly linked to alpha.

Alpha and beta are thought of as constructive concepts when they are illustrative concepts. Unless one is happy to invest over a sufficiently long horizon.

Alpha can be negative even as returns are positive and outperforming the market.

Alpha and Betas are convenient language for active risk and passive risk as long as we don't take them too seriously.