# Liquidity: Volatility Analogs, How to Compare Hedge Funds With Different Redemption Frequencies

In 2008, investors quickly became acquainted with the concept of liquidity, or the lack of it. But what do we mean by liquidity, and how do we price it?

Liquidity premia are already priced in some markets such as the treasury market where the spread between on and off the runs is the implied liquidity premium. But how about markets where you have a single asset and there is no concept of off and on the runs? I would like to look at liquidity from the point of view of option implied volatility in order to understand the relationship between volatility and liquidity in more detail. There are two effects at play which are somewhat inter-related which complicates how we think of the dynamics between volatility and liquidity. I would also like to think of these relationships in a non-mathematical way to avoid the complexities that mathematical rigor would introduce. Let's just get the picture right before we go further.

Volatility tends to rise in liquidity crises. There are two possibilities here. The first is that volatility increases with decreasing liquidity. The second is that the unobserved continuous time volatility is unchanged but that decreased liquidity results in prices being observable with lower frequency (and thus at increasing intervals of time.)

Let us deal with the latter case within an option pricing context. But lets also do it simply. Well, lets at least try.

An option buyer has a right but not an obligation. An option seller or writer has the obligation but not the right. The option writer therefore has to hedge themselves for the possibility that they may have to perform under the obligation. The Black Scholes formula prescribes a hedging strategy, suggesting at any given point in time, the amount of cash (a negative amount, thus borrowing) and underlying instrument in the correct proportions, so that the option obligation is hedged. Since these proportions vary with the cost of borrowing and the underlying price level, there is significant trading involved in maintaining a hedging position. With trading comes trading costs. Even assuming away transaction costs, there will be a profit or loss generated from the trading activity. Hedging an option always generates a trading loss since one is continuously buying high and selling low. Crudely, the more volatile is the underlying, the larger the trading loss. The higher the interest rate, the more costly it is to hold the static hedging position. If one regards the option premium as the cost of hedging it and thus the trading cost plus carrying cost, the higher the interest rate, and the higher the volatility, the higher the price of the option.

Let’s concentrate on the volatility aspect. Volatility is by far the most important factor in pricing an option. So much so that options are often quoted in terms of implied volatility. We know that option prices are an increasing function of the underlying volatility. This is true under all established option pricing models continuous time processes to those which admit discontinuities.

Recall that we assume that the underlying volatility is unchanged but that we now vary the liquidity of the underlying market. This is modeled by varying the interval between transactions and thus price observation. The less liquid is the market, the longer the interval between observations.

The longer the trade interval, the more costly will be the trading cost of hedging since one cannot execute at the prices one wishes to. The cost will vary as an increasing function of the maximum possible gap or jump in the price of the underlying across intervals. It is not dependent on the actual size of the jump or on some probability adjusted size of jump. For the hedger, they must have some idea of the maximum possible jump. Now for a given volatility, the maximum possible jump size is a function of the length of the interval. Luck may result in a price mean reverting within the interval but as one cannot observe it, and one cannot rely on luck, the maximum jump size is an increasing function of the size of the interval.

Thus, the less liquid is a market, the larger the trading intervals, the larger the maximum possible jump and the larger the trading cost in hedging an option on the underlying. The larger the trading cost implies the larger the premium or the price of the option.

We now make a leap in transitioning from the option to underlying by noting that the underlying is a zero strike option on itself, assuming prices can never be negative. Even without this leap we can make our next observation.

Liquidity can be priced in implied volatility spreads. While an absolute level may be hard to obtain, we can obtain some notion of price for an instrument trading under liquid versus illiquid conditions. The details I leave to the mathematicians among us.

One can imagine using this methodology to price the value of having monthly versus quarterly liquidity in hedge funds for example. Already volatility is a factor for sorting and ranking hedge funds in a mean variance framework. This is the Sharpe Ratio. One can think of a Liquidity Adjusted Volatility leading to a Liquidity Risk Ratio which measures Return over Risk free divided by Liquidity Adjusted Volatility. One simple application of this methodology is in pricing liquidity in different liquidity classes of the same fund. Since returns and volatility are the same, one would like to decide whether to invest in the less liquid class in return for a discount in fees. Now you can quantify it.